

Mathematics & Decision Graduate Program

ABOUT THE PHD PROGRAM

This Program is designed to equip national and international students with advanced mathematical skills and decision-making capabilities essential for addressing complex problems in various industries and research fields. Our interdisciplinary curriculum bridges theoretical mathematics and practical decision sciences, preparing graduates to apply mathematical models and analytical techniques to real-world scenarios. Primary objectives of this Program are to:

- Develop a deep understanding of advanced mathematical concepts and their applications.
- Enhance analytical and problem-solving skills.
- Equip students with the ability to formulate and solve complex decision-making problems.
- Prepare graduates for careers in academia, industry, and government sectors.

IMPORTANT DATES

- Application Deadline: 31/07/2024
- Interviews: 15-21/08/2024
- PhD Integration Week: 23-27/09/2024
- PhD Start Date: 01/10/2024

ELIGIBILITY

Applicants should have a strong academic background (Master's degree, Engineering School or equivalent) in related fields such as mathematics, engineering, computer science,

or physics. Previous research experience and a passion for interdisciplinary collaboration are highly desirable.

BENEFITS

- Competitive scholarships and funding opportunities.
- Housing in our campus at Benguerir.
- State-of-the-art research facilities and resources.
- Leading experts and industry partners.
- Research stays opportunities abroad, fostering practical exposure in academia or industry.
- Possibility of joint supervision (*cotutelle*) with International Universities.

CAREER PROSPECTS

Graduates of our program are well-prepared for a variety of career paths, including:

Academia – Academic positions in mathematics, statistics, and related fields.

Industry – Working as data scientists, operations researchers, analysts, and consultants in sectors such as finance, technology, healthcare, energy, and logistics.

Government and Public Policy – Contributing to policy-making and public administration through analytical and data-driven decision-making skills.

WHERE TO APPLY

You can apply via UM6P website following the link.

ABOUT UM6P

The Mohammed VI Polytechnic University (UM6P) is a leading institution in Morocco, renowned for its commitment to innovation, research, and education in science and technology. The university aims to drive economic development in Morocco and Africa by focusing on research areas that address critical global challenges, such as sustainable development, renewable energy, and advanced technologies.



UM6P offers a diverse range of programs across various disciplines, including engineering, agriculture, business, and social sciences, with a strong emphasis on hands-on learning and real-world applications. The university's state-of-the-art facilities and cutting-edge research centers provide students with an environment conducive to academic excellence and innovation. Partnerships with leading global institutions and industries enhance the learning experience, fostering an ecosystem of collaboration and knowledge exchange.

Committed to making a significant impact on society, UM6P places a strong emphasis on sustainable development and social responsibility. The university's initiatives in green energy, smart agriculture, and digital transformation are designed to create solutions that benefit not only Morocco but the entire African continent. By nurturing a new generation of leaders and innovators, UM6P is paving the way for a brighter, more sustainable future.

Through its dynamic ecosystem, UM6P supports students, researchers, and entrepreneurs in transforming ideas into impactful ventures. The university offers various programs and initiatives that provide mentorship, funding, and resources to budding entrepreneurs. Key components of this ecosystem include the UM6P StartGate, an incubator and accelerator that nurtures startups from conception to market entry, and the university's collaboration with industry leaders, which bridges the gap between academic research and practical application.

THE UM6P VANGUARD CENTER

The UM6P Vanguard Center is a pioneering research and innovation hub within the Mohammed VI Polytechnic University, dedicated to addressing contemporary challenges through cutting-edge research and technological advancements. The center focuses on interdisciplinary collaboration, bringing together experts from various fields such as engineering, data science, environmental science, and material science to develop innovative solutions that drive sustainable development and economic growth in Morocco and beyond.

The Vanguard Center provides an environment conducive to groundbreaking research and experimentation. The center's initiatives are designed to foster a culture of innovation, encouraging researchers and students to engage in transformative projects that address critical issues such as renewable energy, water conservation, and smart agriculture. By leveraging advanced technologies like artificial intelligence, machine learning, and digital twins, the Vanguard Center aims to create scalable solutions with a significant impact on society and the environment.



The Vanguard Center also emphasizes strong partnerships with industry leaders, academic institutions, and government agencies to enhance research outcomes and facilitate the practical application of scientific discoveries. Through these collaborations, the center not only advances knowledge but also contributes to the development of skilled professionals who are equipped to lead in various sectors. By bridging the gap between academia and industry, the UM6P Vanguard Center plays a crucial role in driving innovation and fostering sustainable development in the region.

GRADUATE PROGRAM STRUCTURE

Our PhD program is divided into two parts; at the beginning, applicants enroll as PhD Students (Predoc), then become PhD Candidates later;

PhD Student – Still completing coursework and qualifying exams, not yet focused exclusively on dissertation re-

search. Depending on the applicant's background, and upon validation by the Doctoral Committee, the Predoc stage can be reduced to 1 year.

PhD Candidate – Has completed coursework and exams, focusing on dissertation research and writing.



Coursework – PhD students must take courses within our Graduate program, which are divided into:

- Basic Courses
- Fundamental Courses
- Advanced Courses
- Specialized Courses

Qualifying Exam – All students must undergo the qualifying exam before the end of the second year. Each student is required to complete

- ✓ 4 basic courses,
- ✓ 4 fundamental courses,
- ✓ 1 advanced courses,

The written qualifying examination will be conducted in two sessions every year. The qualifying exam is designed to test a student's comprehensive knowledge in their field of study and readiness to undertake independent research.

Directed Research – Directed Research gives students an opportunity to engage in substantial research activities under a faculty advisor's supervision. It helps students develop their research skills, gain experience in their field, and start working on potential dissertation topics. This research is usually undertaken after the completion of initial coursework and before the student is ready to defend their research proposal and pass the qualifying exams. Students may start Directed Research around March of the first year.

English Language Proficiency Requirements – To ensure that all our PhD candidates can successfully engage with the academic content and participate fully in our research community, proficiency in English is essential. Applicants whose native language is not English are required to demonstrate their English language proficiency through standardized tests. We require that all our PhD candidates be proficient in English, which is not the case for PhD Students. Hence, during the predoc, students must take English courses and ensure to achieve the required scores by the end of their predoc. We accept a minimum score of 90 on the TOEFL iBT, or equivalent.

Waivers and Exemptions – PhD Students may be exempt from passing the TOEFL exam, if they meet one of the following criteria:

- Native English Speakers.
- Previous Education in English.

PhD Students who have demonstrated exceptional performance in their qualifying exams, may benefit from a conditional English waiver if their TOEFL score is less than 90 (but higher than 80). In which case, they must take English courses during their candidacy and retake the TOEFL exam within the first year of their candidacy.

Requirements before defending a PhD –

- ★ Coursework
 - Basic Courses (at least 4)
 - Fundamental Courses (at least 4)
 - Advanced Courses (at least 3)
 - Specialized Courses (at least 4)
- ★ Research Credits
 - Research Seminars/Colloquia
 - Directed Research
 - Doctoral Thesis
- ★ Soft Skills & Professional Development
 - Soft Skills
 - Professional Development or Teaching

Remarks –

- 1 credit unit represents one hour of classroom instruction and two hours of outside study per week, during 15 weeks.
- The list of Advanced courses may be updated depending on new research projects.
- Specialized courses are related to ongoing research projects and may be updated in the future.

CURRICULUM STRUCTURE

Our PhD program in Mathematics & Decision at UM6P offers specialized tracks designed to equip students with advanced knowledge and research skills in critical areas of modern science and technology. Our majors include:

Data-Science & Operations Research – This major combines the principles of data science and operations research to develop robust analytical methods for decision-making and optimization in various industries. Students will gain expertise in statistical modeling, machine learning, optimization algorithms, and big data analytics, enabling them to solve complex operational problems.

Mathematical Modeling & Simulation – Focused on the creation and analysis of mathematical models, this track equips students with the skills to simulate real-world phenomena and predict system behaviors. Areas of study include differential equations, numerical methods, and computational simulations, fostering the ability to tackle a wide range of scientific and engineering challenges.

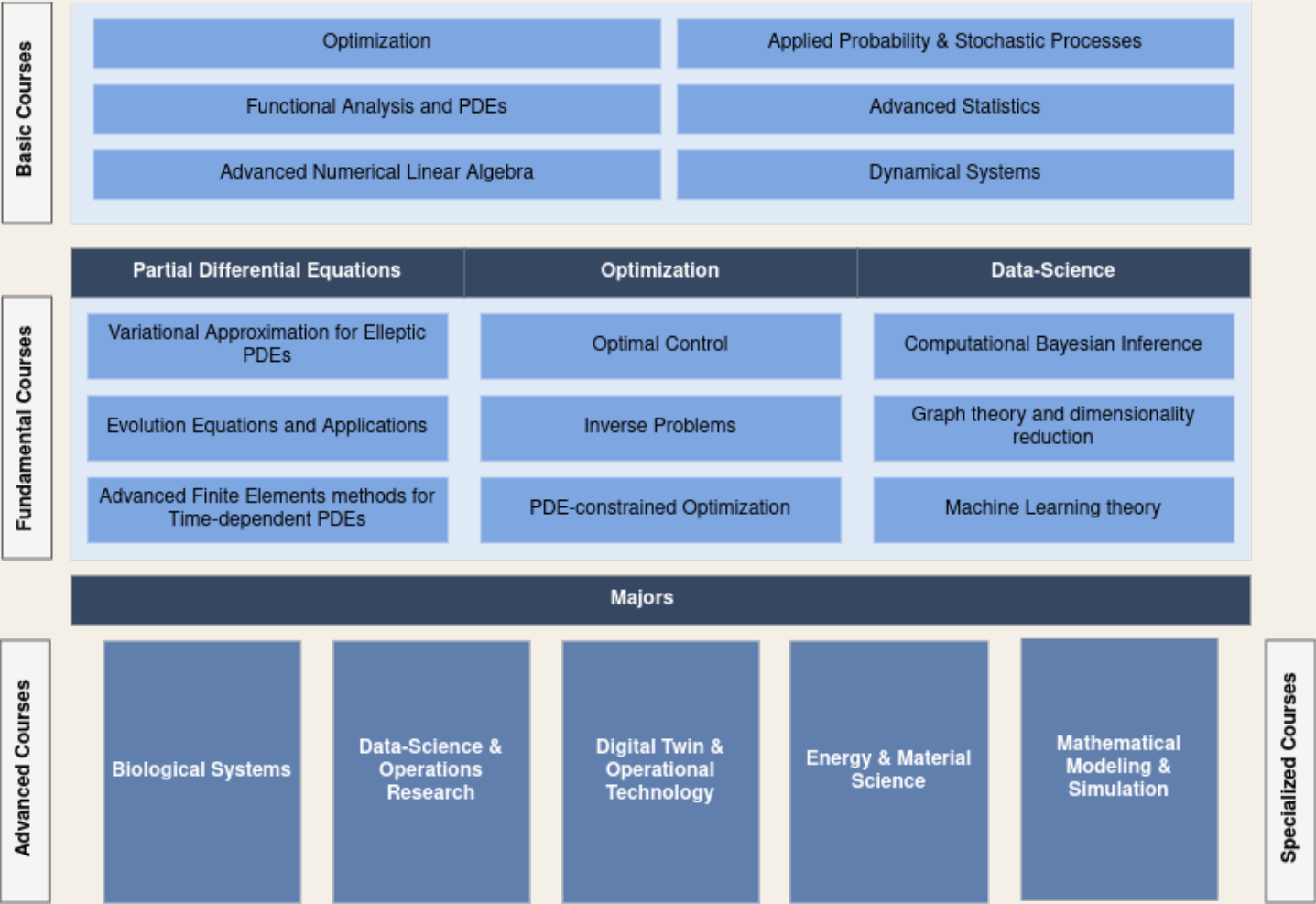
Digital Twin & Operational Technology – This major integrates the concepts of digital twins and operational

technology to enhance the efficiency and performance of complex systems. Students will learn to create virtual models of physical systems for real-time monitoring and optimization, leveraging advanced simulation and control techniques.

Energy & Material Science – Concentrating on the intersection of energy systems and material science, this track prepares students to innovate in sustainable energy solutions and advanced material development. Coursework includes the study of thermodynamics, material properties, and energy conversion processes, with an emphasis on cutting-edge research.

Biological Systems – This track focuses on the application of mathematical and computational methods to understand and model biological processes. Students will explore topics such as systems biology, bioinformatics, and ecological modeling, preparing them to address complex biological challenges through quantitative analysis.

Our program emphasizes interdisciplinary collaboration, innovative research, and practical problem-solving skills, ensuring that graduates are well-prepared to become leaders in academia, industry, and government sectors.



Advanced Numerical Linear Algebra

The Advanced Numerical Linear Algebra course offers a comprehensive introduction to numerical methods for solving linear algebra problems, focusing on algorithms essential for handling systems of linear equations and matrix eigenvalue problems. The course is designed to provide students with classical and iterative methods suitable for large-scale problems, particularly those arising from the discretization of PDEs such as Stokes and Navier-Stokes equations. A basic understanding of elementary linear algebra and foundational programming knowledge is required, though familiarity with high-level programming languages like C or Python is beneficial.

Course Content.

Matrix Analysis and Linear Systems: Covering the fundamental concepts and numerical methods for solving linear and nonlinear systems, eigenvalue problems and computation of matrix functions.

Matrix Factorisation Methods: QR, LU, Singular Value Decomposition (SVD) and its generalisations.

Direct and Iterative Solution Methods: Exploring various algorithms for direct solutions and iterative methods tailored for large-scale systems.

Preconditioning techniques: to improve the convergence of iterative methods such as GMRES, CG or Lanczos type methods by transforming the linear system into one that has more favorable properties.

Sparse Matrix Techniques: Efficient storage and computation methods for sparse matrices; Compressed Sparse Row (CSR) and Compressed Sparse Column (CSC) formats.

Multigrid Methods: Introducing multigrid techniques for efficiently solving large linear systems.

Parallel and Distributed Computing: Algorithms for high-performance computing architectures dividing computations among multiple processors.

CREDITS: 3

CATEGORY: Basic Course

Functional Analysis for PDEs

Functional analysis is essential in various fields of applied science, including quantum mechanics and signal processing, providing powerful tools for modeling and solving complex physical and engineering problems. This course focuses on modern functional analytical tools while avoiding overly difficult prerequisites, making it accessible to a broad audience. Key topics include Hilbertian analysis, spectral theory, sesquilinear/bilinear forms, and weak derivatives, all of which are applied to study prototype models of PDEs such as elliptic, parabolic, and hyperbolic equations.

Course Content.

Hilbertian Analysis Tools: This includes Hilbert bases, spectral theory of compact operators, sesquilinear/bilinear forms, operators defined by these forms, and the Lax-Milgram theorem.

Distributions and Sobolev Spaces: Students will explore test functions, density in L^p -spaces, weak derivatives, Sobolev spaces, and the Poincaré inequality.

Elliptic PDEs: Focus on the existence, uniqueness, and regularity of weak solutions, variational formulation, and energy estimates.

Parabolic PDEs: Study the heat equation, initial value problems, generalized solutions, existence, uniqueness, smoothing effects, and energy estimates for semilinear equations.

Hyperbolic PDEs and Schrödinger Equation: Addressing the wave equation and Schrödinger equation, this section covers energy conservation, existence, uniqueness, and regularity of generalized solutions, along with energy estimates.

CREDITS: 3

CATEGORY: Basic Course

Optimization

This course is designed to provide both theoretical and numerical insights into linear and nonlinear optimization. Students will first be introduced to classical optimization methods, including pattern search, evolutionary-based, and gradient methods, covering the foundational theoretical aspects. As the course progresses, advanced algorithms in nonlinear programming and multi-objective optimization will be explored. A significant emphasis is placed on the practical implementation of these algorithms using Python, enabling students to apply theoretical concepts to real-world problems.

Course Content.

Introduction to Optimization Problems: Overview of major problem families and solving classes in optimization.

Gradient-Free Methods: Techniques such as pattern search methods and evolutionary-based methods, including particle swarms, with practical examples and applications.

Continuous Optimization: Concepts of convexity, differentials, and optimality conditions, with an analysis of the convergence properties of gradient-type methods.

Constrained Optimization: Techniques including penalization methods, projected gradient methods, Lagrangian and duality theory.

Advanced Algorithms: Exploration of advanced methods such as conjugate and nonlinear conjugate gradients, Newton and quasi-Newton methods, spectral gradient methods, and Uzawa algorithms.

CREDITS: 3

CATEGORY: Basic Course

Dynamical Systems

Designed for PhD students, this course delves into the mathematical structures and behaviors of systems that evolve over time, covering both deterministic and stochastic systems. The curriculum spans from foundational concepts to advanced topics, equipping students with the skills necessary to analyze and interpret complex dynamical behaviors in various scientific and engineering contexts.

Course Content.

Introduction: The course starts with fundamental concepts of dynamical systems, including phase space, trajectories, and stability. Students will learn about fixed points, periodic orbits, and the local and global behavior of systems through linear and nonlinear differential equations. The focus will be on understanding the qualitative behavior of dynamical systems rather than merely solving equations.

Linear Systems: the course covers matrix exponentials and Jordan canonical form, providing tools to solve linear differential equations and analyze stability. For nonlinear systems, students will explore linearization techniques, Lyapunov functions, and the Hartman-Grobman theorem to understand local dynamics near critical points.

Chaos theory: including bifurcation theory, strange attractors, and fractals. Students will examine how small changes in initial conditions can lead to vastly different outcomes, a hallmark of chaotic systems. The course also covers the Lorenz system and other classical examples of chaos.

Hamiltonian systems and ergodic theory: providing a bridge between classical mechanics and statistical physics. Students will explore symplectic geometry and action-angle coordinates, and understand the principles of energy conservation and phase space volume preservation.

Stochastic Dynamical Systems: , the course introduces stochastic differential equations (SDEs) and their applications. Students will learn about Brownian motion, Ito calculus, and the Fokker-Planck equation, equipping them with the tools to model and analyze systems influenced by random processes.

Applications: case studies in fields such as population dynamics, epidemiology, climate modeling, and engineering. These real-world examples illustrate how dynamical systems theory can be used to solve practical problems and predict system behaviors.

CREDITS: 3

CATEGORY: Basic Course

Applied Probability and Stochastic Calculus

This course covers the essential concepts of probability theory, stochastic processes, and stochastic calculus, equipping students with the theoretical and practical tools necessary to model and analyze systems affected by uncertainty. The curriculum spans fundamental principles to advanced topics, with applications in finance, physics, biology, and engineering.

Course Content.

Review of Probability Theory: focusing on the foundational concepts needed for more advanced topics. Students will explore probability spaces, random variables, expectation, and convergence theorems, ensuring a solid grounding in the basics of probability.

Stochastic Processes: covering key types such as Poisson processes, Markov chains, and Martingales. Students will learn how these processes are used to model random systems and explore their properties and behaviors through various examples and applications.

Stochastic calculus: with an emphasis on Brownian motion and its properties. The course covers the construction of Brownian motion and explores its role as a fundamental building block in stochastic modeling. Students will delve into Ito calculus, learning to compute Ito integrals and applying Ito's lemma to solve stochastic differential equations (SDEs).

Advanced Topics: such as the Fokker-Planck equation and Girsanov's theorem, providing students with the tools to analyze the behavior of stochastic systems under different conditions and transformations.

Applications: case studies in finance, including the Black-Scholes model for option pricing, and in engineering, with examples from control theory and signal processing.

CREDITS: 3

CATEGORY: Basic Course

Advanced Statistics

This course covers theoretical foundations, advanced methodologies, and practical applications of statistics, equipping students with the skills necessary to conduct high-level research and data analysis. Topics include probability theory, statistical inference, regression models, multivariate analysis, Bayesian statistics, and modern statistical computing techniques.

Course Content.

statistical inference: focusing on both point and interval estimation methods. Students will learn about methods of moments and maximum likelihood estimation for point estimation, and how to construct confidence intervals for means, variances, and proportions. Hypothesis testing, including parametric and non-parametric tests and power analysis, will also be covered in depth.

Regression models: with an emphasis on linear regression, including simple and multiple regression models. The course also covers generalized linear models (GLM) such as logistic and Poisson regression, providing a comprehensive understanding of these powerful tools for data analysis.

Multivariate analysis: starting with multivariate distributions and their properties. The course includes practical techniques such as Principal Component Analysis (PCA) for dimension reduction and exploratory factor analysis, along with clustering methods like hierarchical and k-means clustering.

Bayesian statistics: , students will be introduced to Bayesian inference, including Bayes' theorem and the formulation of prior and posterior distributions. The course will also cover Markov Chain Monte Carlo (MCMC) methods, including Gibbs sampling and the Metropolis-Hastings algorithm, essential for modern Bayesian computation.

Modern statistical computing techniques: with a focus on the use of statistical software such as R and Python. Students will learn resampling methods like bootstrap and permutation tests, and techniques for high-dimensional data analysis, including Lasso and Ridge regression, which are critical for handling large datasets.

Applications: are demonstrated through case studies in fields such as genomics, environmental science, and social sciences. The course includes practical data analysis projects using real-world datasets, allowing students to apply their knowledge in meaningful ways.

CREDITS: 3

CATEGORY: Basic Course

Variational Approximation for Elliptic PDEs

This course covers fundamental concepts such as weak formulation and numerical approximation of PDEs using both finite element methods and mixed finite element methods. Students will explore both coercive and non-coercive problems for scalar and vector-valued PDEs. This course is ideal for graduate students in mathematics, engineering, or related fields who have basic knowledge in analysis, linear algebra, and numerical methods.

Course Content.

Mathematical Preliminaries: Includes distributions, Sobolev spaces, traces, Green's formulas, and the concept of weak formulation. Students will also study well-posedness results through the Lax-Milgram and Banach-Nečas-Babuška theorems and model problems such as Laplace and Stokes equations.

Galerkin Approximation in Banach Spaces: Focus on Galerkin and Petrov-Galerkin methods, discrete well-posedness results, and error analysis through Céa's lemma and Babuška's lemma.

Coercive Problems: Study and approximation of scalar elliptic problems, including the Laplace problem using Lagrange finite elements and spectral methods. This section also covers vector-valued coercive PDEs, like defined Maxwell's equations.

Mixed Finite Element Approximation: Addresses saddle point problems, the Babuška-Brezzi theorem, and solutions to scalar and vector Poisson problems and the Stokes problem in mixed formulation.

Non-coercive PDEs: Covers the Helmholtz problem and first-order PDEs, with methods for Galerkin/least squares approximation.

CREDITS: 3

CATEGORY: Fundamental Course

Advanced Finite Elements Methods for Time-Dependent PDEs

This course delves into finite element methods specifically tailored for time-dependent Partial Differential Equations (PDEs). Students will explore both spatial and temporal discretization techniques for parabolic and hyperbolic equations, using conforming and non-conforming finite elements as well as mixed finite element methods. The curriculum is designed for graduate students in mathematics, engineering, or related fields, who possess foundational knowledge in analysis, linear algebra, numerical methods, and basic finite element methods.

Course Content.

Non-conforming Finite Element Methods: Addressing variational crimes, non-conforming Lagrange finite elements on triangles, and weak enforcement of Dirichlet boundary conditions, along with an introduction to discontinuous Galerkin methods.

Parabolic Problems: Focusing on the heat equation, abstract parabolic problems, weak formulation, and well-posedness. Students will learn semi-discretization in space and various time discretization methods such as Explicit/Implicit Euler and Runge-Kutta schemes, with applications to finite element approximations.

Second-Order Evolution Problems: Covering the wave equation, abstract second-order problems in time, and semi-discretization in space with the Crank-Nicolson scheme.

First-Order PDEs and Maxwell's Equations: Exploring Friedrichs' systems, space semi-discretization, time discretization for conservation equations, and the variational formulation for Maxwell's equations, including the construction of finite element spaces and the Leap Frog scheme for time discretization.

CREDITS: 3

CATEGORY: Fundamental Course

Evolution Equations and Applications

The course deals the study of how states evolve over time within dynamical systems governed by differential equations. These evolution equations are pivotal in various scientific fields, including physics, biology, and finance. This course provides an in-depth exploration of C_0 -semigroups and their applications to evolution equations, emphasizing the theoretical and practical methods used to analyze linear partial differential equations (PDEs) and dynamical systems. Students will gain a thorough understanding of the generation of semigroups, their properties, and their applications to various types of evolution equations, such as parabolic and hyperbolic equations. The course strikes a balance between theoretical concepts and practical applications, equipping students with the skills to analyze and solve complex dynamical problems.

Course Content.

Introduction to Semigroups: defining semigroups and providing examples to illustrate their importance in the study of dynamical systems. Students will learn about the concept of strong continuity and the role of infinitesimal generators in the formation of semigroups. These foundational concepts set the stage for understanding more complex aspects of semigroup theory.

Generation Results: Hille-Yosida Theorem and the Lumer-Phillips Theorem, providing rigorous proofs and practical examples. These theorems are crucial for understanding the conditions under which a semigroup can be generated. Students will explore various examples to solidify their understanding of these fundamental results.

Properties of C_0 -Semigroups: including exponential boundedness, which ensures that the semigroup does not grow too rapidly. The course also examines compact semigroups, which have important applications in functional analysis, and discusses the differentiability and analyticity of semigroups, providing tools for analyzing the smoothness and regularity of solutions to evolution equations.

Spectral Properties and Asymptotic Behaviors: spectral properties of semigroups, focusing on how these properties influence the long-term behavior of solutions. The course covers the analysis of the spectrum of the generator and how it relates to the stability and asymptotic behavior of the semigroup. Understanding these spectral properties is key to predicting how systems evolve over time.

Perturbation of Generators: The Kato-Voigt perturbation theorem is introduced in this chapter, providing a framework for analyzing how small changes in the generator can affect the semigroup. This theorem is particularly useful for studying the stability and robustness of solutions to evolution equations under perturbations.

Evolution Equations: abstract Cauchy problems and inhomogeneous evolution equations. Students will learn about the well-posedness and regularity of solutions, which are critical for ensuring that the solutions to these problems are unique and stable under small perturbations. The course provides a detailed analysis of the conditions required for well-posedness and the techniques used to establish regularity.

Parabolic Equations: The heat equation is studied as a primary example of a parabolic equation. Students will explore the semigroup approach to solving the heat equation, emphasizing maximum principles and the long-term behavior of solutions. This chapter provides practical techniques for analyzing and solving parabolic PDEs, which are common in heat transfer and diffusion processes.

Hyperbolic Equations: focusing on the wave equation. Students will learn about energy methods and the semigroup approach to solving hyperbolic equations. The course also covers D'Alembert's formula, providing a concrete example of how semigroups can be used to solve and analyze wave equations, which are prevalent in the study of oscillatory phenomena and wave propagation.

CREDITS: 3

CATEGORY: Fundamental Course

Inverse Problems: Theory and Numerical Methods

This course explores the intriguing field of inverse problems, which involves determining unknown causes from observed effects using a set of measurements. These problems are crucial in various scientific and engineering disciplines, including seismology, medical imaging, and astronomy. The course covers the mathematical formulation of inverse problems, their conditional well-posedness, and qualitative properties such as existence, uniqueness, and stability. Students will also learn various methods and techniques for solving inverse problems and approximating their solutions, with a focus on both theoretical foundations and practical applications.

Course Content.

Introduction and Examples: Overview of inverse problems with real-world applications.

Linear Inverse and Ill-posed Problems: Understanding the challenges associated with ill-posed problems.

Theoretical Methods: Techniques to prove uniqueness and stability, such as Carleman estimates.

Regularization Strategies: Including Tikhonov regularization to stabilize solutions.

Numerical Methods: Techniques such as singular value decomposition, quasi-solution approaches, gradient calculation using the adjoint state method, and iterative algorithms like Landweber iteration and Conjugate Gradient algorithms.

CREDITS: 3

CATEGORY: Fundamental Course

Optimal Control: Theory and Applications

In this course, we explore optimal control strategies for dynamic systems, focusing on minimizing or maximizing performance functions. Originating from the calculus of variations, the field of optimal control became an independent discipline in the early 1950s thanks to significant contributions such as Pontryagin's Maximum Principle and Bellman's Dynamic Programming. This course is designed to cover both theoretical and numerical aspects, providing students with the foundational principles and practical methods for solving optimal control problems across various fields including engineering, economics, biology, and logistics.

Course Content.

Introduction and Examples: An overview of optimal control problems and their applications.

Calculus of Variations: Exploring the Euler-Lagrange equations and necessary conditions for optimality. Pontryagin's Maximum Principle:] Applications to linear and nonlinear systems.

Dynamic Programming: Understanding Bellman's Principle of Optimality and the Hamilton-Jacobi-Bellman (HJB) equation.

Linear Quadratic Regulator (LQR): Studying this specific optimal control problem.

Applications and Numerical Methods: Practical methods for solving and approximating solutions to optimal control problems.

CREDITS: 3

CATEGORY: Fundamental Course

PDE-Constrained Optimization

The course is focused on the theory and methods for solving optimization problems constrained by partial differential equations (PDEs). This course addresses the formulation and solution of optimization problems where state variables are governed by PDEs, integrating theoretical insights, numerical techniques, and practical applications in fields such as fluid dynamics, structural optimization, and control theory. The course is designed for students with a background in advanced calculus, Hilbert spaces, linear algebra, and basic knowledge in PDEs and optimization.

Course Content.

Introduction to PDE-Constrained Optimization: Fundamental concepts and an overview of the field.

Mathematical Preliminaries: Detailed exploration of differentiability in Banach and Hilbert spaces, and state differentiability for elliptic equations.

Optimality Conditions: Conditions necessary for achieving optimal solutions in PDE-constrained settings.

Numerical Methods for PDEs: An overview of the numerical techniques used to solve PDEs.

Numerical Optimization Techniques: Overview of optimization methods applicable to PDE-constrained problems.

Solving PDE-Constrained Optimization Problems: Examination of different approaches such as discretize-then-optimize versus optimize-then-discretize, and the implementation of adjoint methods.

Applications and Case Studies: Practical examples and case studies from various fields.

Advanced Topics and Research Frontiers: Exploration of cutting-edge research and advanced topics in the field.

CREDITS: 3

CATEGORY: Fundamental Course

Graph Theory and Dimensionality Reduction

The "Graph Theory and Dimensionality Reduction" course provides an in-depth study of complex networks and the mathematical tools necessary for their analysis and simplification. With applications spanning sociology, biology, engineering, information sciences, economics, psychology, and more, this course equips students with the skills to understand, control, and improve systems represented by large complex networks. The course covers essential graph theory concepts, numerical linear algebra tools, and various dimensionality reduction techniques, all of which are critical for handling high-dimensional data in real-world applications.

Course Content.

Numerical Linear Algebra Tools: This chapter focuses on foundational and advanced linear algebra techniques essential for understanding and implementing graph theory and dimensionality reduction methods. **Basic Linear Algebra Background:** Review of vectors, matrices, and basic operations. **Singular Value Decomposition (SVD):** Techniques for decomposing matrices, with applications in data compression and noise reduction. **QR Factorization and Least-Squares Problems:** Solving linear systems and optimization problems. **Fast Fourier Transform (FFT):** Transform techniques for signal processing. **Krylov Subspace Methods:** Efficient iterative methods for solving large linear systems. **Tensors for Multi-linear Algebra Problems:** Introduction to tensor operations and their applications.

Graphs and Network Analysis: This chapter introduces the fundamental concepts of graph theory and their applications in analyzing complex networks. **Introduction to Graph Theory:** Basic definitions and types of graphs. **Fundamental Concepts and Terminology:** Nodes, edges, paths, cycles, connectivity, and graph isomorphism. **Graph Clustering:** Techniques for identifying communities within networks. **Graph Embedding:** Methods for representing graphs in lower-dimensional spaces. **Krylov-Type Subspace Methods for Graphs:** Application of Krylov methods to graph problems.

Dimensionality Reduction Methods: This chapter explores various techniques for reducing the dimensionality of data, making it easier to analyze and interpret. **Principal Component Analysis (PCA):** Reducing dimensionality while preserving variance. **Linear Discriminant Analysis (LDA):** Techniques for feature extraction and classification. **Laplacian Eigenmaps:** Non-linear dimensionality reduction using spectral techniques. **Locally Linear Embedding (LLE):** Capturing the local geometry of data. **Locality Preserving Projections (LPP):** Preserving local structure in data. **Orthogonal Neighborhood Preserving Projection (ONPP):** Maintaining neighborhood relations. **Kernel Methods:** Advanced techniques for non-linear data.

Sparse Compressive Sensing: Introduction to sparse signal representation. **Computational Methods:** Techniques for solving compressive sensing problems. **Robust Principal Component Analysis (RPCA):** Methods for dealing with noise and outliers. **Least Absolute Shrinkage and Selection Operator (Lasso):** Techniques for sparse regression and feature selection.

Matrix and Tensor Completion: **The Mathematical Problem of Completion:** Understanding the fundamentals. **The Alternating Least Squares (ALS):** Techniques for solving matrix completion problems. **The Singular Value Thresholding (SVT):** Methods for low-rank matrix recovery. **The Stochastic Gradient Descent (SGD) Method:** Techniques for optimizing large-scale problems.

Applications: This chapter focuses on real-world applications of graph theory and dimensionality reduction methods. **Image, Signal, and Video Processing:** Applications in enhancing and compressing media. **Classification, Detection, and Restoration:** Techniques for data analysis and interpretation. **Hyperspectral Images:** Advanced methods for processing and analyzing high-dimensional image data. **Face and Object Recognition:** Improving recognition systems through advanced mathematical techniques.

CREDITS: 3

CATEGORY: Fundamental Course

Machine Learning Theory

"Machine Learning Theory" is a comprehensive course designed for graduate students seeking a deep understanding of the theoretical foundations of machine learning. This course covers fundamental concepts, key algorithms, and theoretical frameworks that underpin modern machine learning techniques. Emphasis is placed on understanding the principles behind machine learning models, the mathematical tools used to analyze them, and the theoretical guarantees of their performance. The course balances rigorous theoretical analysis with practical considerations, preparing students for advanced research and application in machine learning.

Course Content.

- Introduction to Machine Learning:** overview of machine learning, defining its scope and relevance in various fields. Students will explore different types of learning, including supervised, unsupervised, and reinforcement learning. The introductory segment sets the stage for a deeper dive into the theoretical aspects of machine learning.
- Mathematical Foundations:** Topics include probability theory, linear algebra, and optimization techniques. Students will learn about important concepts such as probability distributions, eigenvalues and eigenvectors, convex optimization, and gradient-based methods. These tools are crucial for understanding the underlying mechanics of machine learning algorithms.
- Learning Theory:** PAC (Probably Approximately Correct) learning framework, VC (Vapnik-Chervonenkis) dimension, and Rademacher complexity. These concepts help in understanding the capacity of different learning models and their ability to generalize from training data to unseen data.
- Supervised Learning Algorithms:** linear regression, logistic regression, and support vector machines (SVM). The course emphasizes the theoretical properties of these algorithms, including consistency, convergence rates, and robustness. Students will analyze the theoretical guarantees provided by these models and understand the conditions under which they perform well.
- Unsupervised Learning and Clustering:** k-means, hierarchical clustering, and Gaussian mixture models. The course will cover theoretical aspects like the objective functions these algorithms optimize and their convergence properties. Students will also learn about dimensionality reduction techniques such as PCA (Principal Component Analysis) and their theoretical underpinnings.
- Neural Networks and Deep Learning:** Topics include the universal approximation theorem, backpropagation algorithm, and various architectures like CNNs (Convolutional Neural Networks) and RNNs (Recurrent Neural Networks). The course also discusses the challenges in training deep networks, such as vanishing and exploding gradients, and the theoretical insights into why deep learning works.
- Generalization and Regularization:** The course explores the concepts of overfitting and underfitting, and the techniques used to mitigate these issues. Students will learn about regularization methods such as L1 and L2 regularization, dropout, and early stopping. The theoretical analysis of these techniques will help students understand how they contribute to the generalization ability of machine learning models.
- Advanced Topics and Current Research:** kernel methods, ensemble learning, and Bayesian methods. Students will explore the theoretical basis of these methods and their applications. The course also discusses current research trends and open problems in machine learning theory, preparing students for cutting-edge research in the field.

CREDITS: 3

CATEGORY: Fundamental Course

Computational Bayesian Inference

This course focuses on the principles and computational techniques of Bayesian inference. This course provides a comprehensive understanding of Bayesian methods for statistical analysis, emphasizing the development and implementation of computational algorithms for Bayesian modeling and inference. Students will explore various applications across different domains, including machine learning, data science, and engineering.

Course Content.

Introduction to Bayesian Inference: fundamental concepts of Bayesian inference, including Bayes' theorem, prior and posterior distributions, and the concept of likelihood. Students will learn about the advantages of the Bayesian approach compared to frequentist methods and explore basic examples of Bayesian inference.

Bayesian Modeling: the construction of Bayesian models, covering topics such as conjugate priors, hierarchical models, and model selection. The chapter emphasizes the importance of choosing appropriate priors and constructing flexible models that can capture the underlying structure of data.

Markov Chain Monte Carlo (MCMC) Methods: Metropolis-Hastings algorithm, Gibbs sampling, and Hamiltonian Monte Carlo (HMC). The course will cover the theoretical foundations of these methods as well as practical implementation using Python and R.

Variational Inference: Topics include mean-field approximation, coordinate ascent variational inference (CAVI), and stochastic variational inference (SVI). The chapter emphasizes the trade-offs between variational methods and MCMC in terms of computational efficiency and accuracy.

Bayesian Nonparametrics: focusing on Dirichlet processes, Gaussian processes, and other flexible modeling approaches that allow for an infinite-dimensional parameter space. Students will learn about the applications of these methods in clustering, regression, and other tasks.

Bayesian Model Averaging and Model Selection: Bayes factors, posterior predictive checks, and Bayesian model averaging (BMA). The chapter emphasizes the importance of model uncertainty and the benefits of incorporating it into the inference process.

Advanced Topics in Bayesian Computation: sequential Monte Carlo (SMC) methods, particle filtering, and approximate Bayesian computation (ABC). Students will explore the applications of these methods in dynamic systems and complex models.

Applications and Case Studies: machine learning, bioinformatics, finance, and engineering. Students will work on projects that apply Bayesian methods to real-world data, reinforcing the concepts learned throughout the course.

CREDITS: 3

CATEGORY: Fundamental Course

Model Order Reduction Methods for Large Problems

This course is designed to equip students with advanced techniques in computational science and engineering for reducing the complexity of mathematical models while preserving their essential features and dynamics. Model Order Reduction (MOR) is particularly valuable for systems described by high-dimensional differential equations, such as those in control systems, circuit simulation, fluid dynamics, structural mechanics, and various other fields. The course aims to develop reduced-order, computationally efficient models that retain the essential behavior and properties of the original systems.

Course Content.

Introduction to Model Order Reduction: Overview of MOR techniques, their importance, and applications across various fields. Differentiating between linear and nonlinear model order reduction methods.

Numerical Linear Algebra Tools: Essential linear algebra methods that form the backbone of MOR techniques. Focus on matrix factorizations, eigenvalue problems, and iterative solvers.

System Norms and Gramians: H_2 and H_∞ norms: Understanding and calculating these norms. Controllability and observability Gramians: Their role in system analysis and MOR.

Balanced Truncation: Techniques for reducing model order while preserving stability and important system properties. Detailed analysis of the balanced truncation method and its computational aspects.

Projection-Based Methods: Krylov subspace methods: Efficient techniques for projection in high-dimensional spaces. Proper Orthogonal Decomposition (POD): Methods for decomposing and reducing models based on orthogonality.

Hankel Norm Approximation: Approaches for approximating models using Hankel norms. Comparison with other norm-based reduction methods.

Moment Matching and Rational Approximation: Techniques for approximating models by matching moments and using rational functions. Introduction to tangential interpolation methods.

Data-Driven and Machine Learning Approaches: Integration of machine learning techniques for model reduction. Empirical approximation methods and their practical applications.

Numerical Methods for Large-Scale Equations: Solving large Lyapunov and Riccati equations efficiently. Application of these methods in MOR.

Applications in Various Fields: Practical applications of MOR techniques in control theory, fluid dynamics, and structural analysis. Case studies demonstrating the effectiveness of MOR in real-world problems.

CREDITS: 2

CATEGORY: Advanced Course

Advanced Signal Processing

This course provides an in-depth exploration of advanced signal processing techniques. Topics include time-frequency analysis, wavelet transforms, adaptive filtering, and statistical signal processing. Students will learn to design and implement algorithms for signal enhancement, detection, and estimation in various applications such as communications, radar, and biomedical engineering.

Course Content.

Time-Frequency Analysis: Introduction to time-frequency representations. Short-time Fourier transform (STFT). Wigner-Ville distribution and spectrograms. Applications in signal analysis and processing.

Wavelet Transforms: Introduction to wavelets and multi-resolution analysis. Continuous and discrete wavelet transforms. Wavelet-based signal processing techniques. Applications in denoising, compression, and feature extraction.

Adaptive Filtering: Fundamentals of adaptive filters. Least mean squares (LMS) and recursive least squares (RLS) algorithms. Adaptive noise cancellation and echo cancellation. Applications in communications and audio processing.

Statistical Signal Processing: Random processes and spectral estimation. Parametric and non-parametric methods. Detection and estimation theory. Applications in radar, sonar, and biomedical signal processing.

CREDITS: 2

CATEGORY: Advanced Course

Digital Image Processing

This course covers the principles and techniques of digital image processing. Topics include image enhancement, restoration, segmentation, and compression. Students will learn about various algorithms and their applications in areas such as medical imaging, remote sensing, and computer vision.

Course Content.

Image Enhancement Techniques: Point processing methods: contrast stretching, histogram equalization. Spatial filtering: smoothing and sharpening. Frequency domain techniques: Fourier transform and filtering. Applications in image quality improvement.

Image Restoration: Degradation models and noise reduction. Inverse filtering and Wiener filtering. Regularization methods and blind deconvolution. Applications in removing blur and noise.

Image Segmentation: Edge detection methods: Sobel, Canny. Thresholding and region-based segmentation. Clustering methods: k-means, Gaussian mixture models. Applications in object detection and medical imaging.

Image Compression: Lossless compression: Huffman coding, LZW. Lossy compression: JPEG, wavelet-based methods. Transform coding and quantization. Applications in data storage and transmission.

CREDITS: 2

CATEGORY: Advanced Course

Hyperspectral Imaging

This course focuses on hyperspectral imaging, a technique that captures and processes information across the electromagnetic spectrum. Students will learn about the principles of hyperspectral imaging, data acquisition, preprocessing, and analysis methods. Applications in agriculture, environmental monitoring, and defense will be discussed.

Course Content.

Principles of Hyperspectral Imaging: Introduction to hyperspectral imaging. Spectral signatures and reflectance properties. Hyperspectral sensors and platforms. Data acquisition and calibration.

Hyperspectral Data Acquisition: Sensor design and calibration. Data preprocessing: radiometric correction, atmospheric correction. Noise reduction techniques. Applications in remote sensing and environmental monitoring.

Preprocessing Techniques: Spectral and spatial preprocessing. Dimensionality reduction: PCA, ICA. Feature extraction and selection. Applications in data compression and noise reduction.

Analysis Methods: Spectral unmixing and endmember extraction. Classification algorithms: SVM, neural networks. Anomaly detection and target detection. Applications in agriculture, mineral exploration, and defense.

CREDITS: 2

CATEGORY: Advanced Course

Machine Learning for Signal and Image Processing

This course explores the application of machine learning techniques to signal and image processing. Topics include feature extraction, classification, clustering, and deep learning. Students will learn to apply these techniques to real-world problems in areas such as speech recognition, image classification, and pattern recognition.

Course Content.

Feature Extraction Techniques: Principal component analysis (PCA). Independent component analysis (ICA). Feature selection methods. Applications in signal and image processing.

Classification and Clustering: Supervised learning: SVM, decision trees. Unsupervised learning: k-means, hierarchical clustering. Evaluation metrics for classification and clustering. Applications in speech and image recognition.

Deep Learning for Signal and Image Processing: Convolutional neural networks (CNNs). Recurrent neural networks (RNNs). Autoencoders and generative models. Applications in image classification and speech recognition.

CREDITS: 2

CATEGORY: Advanced Course

Statistical Methods for Image Analysis

This course covers statistical methods used in the analysis of images. Topics include image modeling, Bayesian methods, Markov random fields, and variational methods. Students will learn to develop and implement algorithms for image segmentation, object recognition, and texture analysis.

Course Content.

Image Modeling and Bayesian Methods: Probabilistic models for images. Bayesian inference and estimation. Applications in image reconstruction and denoising.

Markov Random Fields: Introduction to Markov random fields (MRFs). Parameter estimation and learning. Applications in image segmentation and texture analysis.

Variational Methods: Variational principles in image processing. Level set methods for image segmentation. Applications in object recognition and shape analysis.

CREDITS: 2

CATEGORY: Advanced Course

Deep Learning and Generative Models

This course provides students with a comprehensive understanding of deep learning techniques and generative models. The course covers the theoretical foundations, algorithms, and applications of deep learning, including neural networks, convolutional networks, recurrent networks, and advanced generative models such as GANs, VAEs, and autoregressive models.

Course Content.

Introduction to Deep Learning: Students will learn about neural networks, activation functions, and the backpropagation algorithm.

Convolutional Neural Networks (CNNs): Students will explore CNNs, including their architecture, operation, and applications. Topics include convolutional layers, pooling layers, and popular CNN architectures such as AlexNet, VGG, and ResNet.

Recurrent Neural Networks (RNNs): Topics include the architecture of RNNs, LSTMs, GRUs, and applications in natural language processing and time-series analysis.

Advanced Deep Learning Techniques: transfer learning, reinforcement learning, and unsupervised learning. The chapter covers techniques for training deep networks, including optimization algorithms and regularization methods.

Generative Adversarial Networks (GANs): This chapter introduces GANs, including their architecture, operation, and applications. Students will learn about the training process of GANs, different types of GANs, and their applications in image synthesis and data augmentation.

Variational Autoencoders (VAEs): Students will explore VAEs, including their theoretical foundations, architecture, and applications. Topics include the variational inference framework, the reparameterization trick, and applications in generative modeling and representation learning.

Autoregressive Models: This chapter covers autoregressive models such as PixelRNN, PixelCNN, and WaveNet. Students will learn about the architecture and applications of these models in generating sequential data.

Applications and Case Studies: The course concludes with applications and case studies demonstrating the use of deep learning and generative models in various fields. Students will work on projects involving real-world data, reinforcing the concepts learned throughout the course.

CREDITS: 3

CATEGORY: Specialized Course

Deep Reinforcement Learning: Theory and Applications

The course is focused on the principles and techniques of deep reinforcement learning (DRL). The course covers the theoretical foundations, algorithms, and practical applications of DRL, including value-based methods, policy-based methods, and actor-critic methods. Students will learn how to apply DRL to solve complex decision-making problems in various domains.

Course Content.

Introduction to Reinforcement Learning: This chapter introduces the key concepts of reinforcement learning, including the agent-environment interaction, reward signals, and the Markov decision process (MDP). Students will learn about the difference between reinforcement learning and other types of machine learning.

Value-Based Methods: Students will explore value-based methods, including Q-learning and deep Q-networks (DQN). The chapter covers the architecture of DQNs, experience replay, and target networks. Students will implement DQNs to solve simple RL problems.

Policy-Based Methods: Students will learn about the advantages of policy-based methods over value-based methods and implement policy gradient algorithms.

Actor-Critic Methods: The chapter covers the architecture and implementation of actor-critic algorithms, including A3C (Asynchronous Advantage Actor-Critic) and PPO (Proximal Policy Optimization).

Advanced Topics in DRL: This chapter explores advanced topics in DRL, including multi-agent reinforcement learning, hierarchical reinforcement learning, and meta-reinforcement learning. Students will learn about the challenges and solutions in these advanced areas.

Applications of DRL: Students will explore various applications of DRL, including robotics, game playing, and autonomous systems. The chapter covers case studies and real-world applications, highlighting the impact of DRL on different industries.

Practical Implementation of DRL Algorithms: This chapter provides hands-on experience in implementing DRL algorithms using deep learning frameworks. Students will work on projects involving real-world RL problems, reinforcing the concepts learned throughout the course.

CREDITS: 2

CATEGORY: Advanced Course

Pattern Recognition

"Pattern Recognition" is a graduate-level course that focuses on the principles and techniques used to identify patterns and regularities in data. The course covers statistical pattern recognition, machine learning approaches, feature extraction, and classification methods. Students will learn how to apply these techniques to solve real-world problems in various domains such as image recognition, speech processing, and bioinformatics.

Course Content.

Introduction to Pattern Recognition: Students will learn about the history and evolution of pattern recognition and its importance in various fields.

Statistical Pattern Recognition: Bayesian decision theory, maximum likelihood estimation, and the Bayes classifier. The chapter covers parametric and non-parametric methods for pattern recognition.

Feature Extraction and Selection: dimensionality reduction, principal component analysis (PCA), linear discriminant analysis (LDA), and feature selection methods. Students will learn about the importance of selecting relevant features for improving classification performance.

Classification Methods: linear classifiers, k-nearest neighbors (k-NN), support vector machines (SVM), and decision trees. The chapter covers the theory, implementation, and evaluation of these classifiers.

Neural Networks for Pattern Recognition: multilayer perceptrons (MLP), backpropagation, and deep learning techniques for pattern recognition. Students will learn about the advantages and limitations of neural networks in pattern recognition tasks.

Clustering and Unsupervised Learning: k-means clustering, hierarchical clustering, and Gaussian mixture models (GMM). The chapter covers the evaluation of clustering results and applications of clustering in pattern recognition.

Advanced Topics in Pattern Recognition: ensemble methods, boosting, and bagging. Students will learn about the use of ensemble techniques to improve classification performance and robustness.

Applications and Case Studies: The course concludes with applications and case studies demonstrating the use of pattern recognition in various fields. Students will work on projects involving real-world data, reinforcing the concepts learned throughout the course.

CREDITS: 3

CATEGORY: Specialized Course

Data-Driven Models & Control

"Data-Driven Models & Control" is a graduate-level course that focuses on the development and application of data-driven models and control strategies for dynamic systems. The course covers techniques for modeling systems directly from data, designing controllers based on data-driven models, and implementing these methods in real-world applications. Students will explore various machine learning and statistical methods used to create accurate and efficient models for control purposes.

Course Content.

Introduction to Data-Driven Modeling: Key concepts of data-driven modeling and control. Importance of data in modern control systems. Differences between model-based and data-driven approaches. Applications of data-driven models in engineering.

System Identification: Techniques for identifying system dynamics from data. Time-domain and frequency-domain identification. Parametric and non-parametric methods. Optimization techniques for system identification. Practical considerations and challenges in system identification.

Machine Learning for Dynamic Systems: Application of machine learning techniques to model dynamic systems. Supervised learning, regression, and classification algorithms. Neural networks and deep learning methods. Use of machine learning techniques to create accurate and generalizable models for control purposes.

Data-Driven Control Design: Data-driven control design methodologies. Model predictive control (MPC). Reinforcement learning-based control. Adaptive control. Theoretical foundations and practical implementation of control strategies.

Statistical Methods for Control: Use of statistical methods in control design and analysis. Statistical estimation, hypothesis testing, and Bayesian inference. Role of uncertainty and variability in control systems. Accounting for uncertainty in data-driven control design.

Optimization Techniques for Control: Optimization techniques used in data-driven control. Convex and non-convex optimization. Gradient-based methods and evolutionary algorithms. Application of optimization techniques to optimize control performance and robustness.

Real-Time Implementation: Real-time implementation of data-driven models and control strategies. Hardware-in-the-loop simulation. Real-time optimization. Use of embedded systems for control. Practical challenges and solutions in deploying data-driven control systems.

Applications and Case Studies: Applications of data-driven models and control in various fields. Robotics, automotive systems, aerospace, and industrial automation. Projects involving real-world data. Reinforcement of concepts through real-world applications.

CREDITS: 2

CATEGORY: Advanced Course

Integer Programming

"Integer Programming" is a graduate-level course that focuses on the theory, algorithms, and applications of integer programming (IP). The course covers both pure and mixed-integer programming, emphasizing the mathematical foundations, solution techniques, and practical applications in various fields such as logistics, finance, and manufacturing. Students will learn how to model complex optimization problems using integer programming and solve them using advanced algorithms and software tools.

Course Content.

Introduction to Integer Programming: Overview of integer programming. Differences between linear programming and integer programming. Applications of integer programming in various fields.

Formulation of Integer Programming Problems: Formulating integer programming models. Pure vs. mixed-integer programming. Common constraints and objective functions in integer programming.

Linear Relaxation and Bounds: Linear relaxation of integer programming problems. Upper and lower bounds. Gap analysis and its significance in IP.

Branch and Bound Method: Principles of the branch and bound algorithm. Implementation of branch and bound. Strategies for branching and bounding.

Cutting Plane Methods: Introduction to cutting planes. Gomory cuts and other cutting planes. Integrating cutting planes with branch and bound.

Advanced Solution Techniques: Branch and cut method. Branch and price method. Heuristics and metaheuristics for integer programming.

Applications and Case Studies: Integer programming applications in logistics, finance, and manufacturing. Real-world case studies and problem-solving sessions. Projects involving the modeling and solving of complex IP problems.

Software Tools and Implementation: Overview of integer programming software (e.g., CPLEX, Gurobi, Python). Hands-on training with IP solvers. Implementing custom IP algorithms.

Advanced Topics: Stochastic integer programming. Multi-objective integer programming. Robust and fuzzy integer programming.

CREDITS: 2

CATEGORY: Advanced Course

Shape and Topology Optimization in Engineering

This graduate-level course explores the mathematical and computational techniques used for optimizing the shape and topology of structures in engineering applications. The course covers fundamental concepts, mathematical formulations, and numerical methods used in shape and topology optimization. Students will learn how to apply these techniques to design lightweight, efficient, and innovative structures in various engineering fields.

Course Content.

Introduction to Shape and Topology Optimization: Overview of optimization in engineering design. Distinction between shape optimization and topology optimization. Historical development and applications in engineering.

Mathematical Foundations: Students will learn the mathematical principles underlying optimization problems, focusing on the calculus of variations, optimality conditions, and sensitivity analysis. This chapter establishes the theoretical groundwork necessary for understanding and solving shape and topology optimization problems.

Shape Optimization: Parametric and non-parametric shape optimization. Level set methods for shape optimization. Shape derivatives and boundary variation techniques.

Topology Optimization: Introduction to topology optimization problems. SIMP (Solid Isotropic Material with Penalization) method. Level set method for topology optimization. Density-based approaches and homogenization.

Numerical Methods: Finite element methods (FEM) in optimization. Discretization techniques. Solving large-scale optimization problems. Use of gradient-based and gradient-free optimization algorithms.

Computational Tools and Software: Overview of software tools used in shape and topology optimization. Hands-on training with tools such as ANSYS, Abaqus, COMSOL, and Python. Implementing custom optimization algorithms.

Applications in Engineering: Structural optimization in aerospace, automotive, and civil engineering. Optimization of mechanical components and systems. Case studies on optimizing heat exchangers, compliant mechanisms, and biomedical devices.

Advanced Topics: Multi-objective optimization. Robust optimization under uncertainty. Optimization of composite materials and structures. Integration of optimization with additive manufacturing.

Project and Case Studies: Case studies highlighting successful applications of shape and topology optimization in industry.

CREDITS: 2

CATEGORY: Advanced Course

Density Functional Theory (DFT)

The "Density Functional Theory (DFT)" course provides an in-depth introduction to this essential quantum mechanical method used to analyze the electronic structure of many-body systems such as atoms, molecules, and condensed phases. Recognized as a foundational framework in modern theoretical physics, chemistry, and engineering, DFT is particularly useful for studying electronic properties. This course will cover the fundamental theoretical principles and practical applications of DFT, making it a vital tool for students interested in electronic structure analysis.

Course Content.

Introduction to DFT: Overview of the basics and historical development.

The Kohn-Sham Equations: Understanding the core equations that form the backbone of DFT.

Practical Aspects of DFT Calculations: Techniques and tools for performing DFT calculations.

Applications in Quantum Chemistry: Using DFT to solve problems in quantum chemistry.

Applications in Solid-State Physics: Exploring DFT's role in solid-state physics.

DFT Software and Tools: Introduction to various DFT software packages and their applications.

Case Studies and Research Applications: Real-world examples and current research utilizing DFT.

Limitations of DFT and Current Research Directions: Discussing the boundaries of DFT and ongoing research to expand its capabilities.

CREDITS: 2

CATEGORY: Specialized Course

Electrochemical Energy Systems

The "Electrochemical Energy Systems" course provides a comprehensive exploration of electrochemical systems used across various applications, including stationary, portable, and transportation sectors. While these systems offer numerous advantages for specific applications, they also present certain limitations. This course is designed to impart foundational knowledge and practical insights into electrochemical energy systems. Key topics include the thermodynamics and kinetics of electrochemical processes, electrode materials, electrochemical characterization techniques, and different types of batteries, fuel cells, supercapacitors, as well as emerging technologies.

Course Content.

Electrochemical Thermodynamics and Kinetics: Understanding the fundamental principles governing electrochemical reactions.

Electrode Materials: Exploring the properties and applications of various electrode materials in electrochemical systems.

Applications in Batteries: Detailed study of different types of batteries, their design, functionality, and applications.

Emerging Electrochemical Energy Technologies: Introduction to cutting-edge technologies and innovations in the field of electrochemical energy systems.

CREDITS: 2

CATEGORY: Specialized Course

Non-equilibrium Processes in Energy Conversion

This course aims to equip students with a comprehensive understanding of the non-equilibrium processes that occur in various energy conversion systems. By delving into these processes, students will learn how to achieve significant technological advancements and improve the efficiency of energy systems, thereby driving progress in industrial applications and promoting environmental sustainability. The course covers theoretical foundations, modeling techniques, and practical applications, all aimed at enhancing the performance of energy devices.

Course Content.

Introduction to Non-equilibrium Thermodynamics: Fundamental principles and concepts.

Transport Phenomena: Detailed study of heat conduction and mass diffusion processes.

Non-equilibrium Processes in Heat Engines: Analyzing how these processes impact the efficiency and operation of heat engines.

Non-equilibrium Processes in Nanomaterials and Nanostructures: Exploring the unique behaviors and applications in the context of advanced materials.

CREDITS: 2

CATEGORY: Specialized Course

Mathematical Methods for Material Science

The "Mathematical Methods for Material Science" course is designed to provide an interdisciplinary approach integrating analytical and experimental techniques from engineering, physics, and chemistry to understand material properties and behavior across various length scales. This understanding is essential for designing new materials and advancing technological innovations. The course aims to equip students with essential mathematical and numerical methods used in material science, fostering a robust framework for future technological developments. This growing field has fostered a strong partnership between mathematics and materials science, contributing significantly to technological advancements.

Course Content.

Introduction to Mathematical Methods in Material Science: An overview of the role of mathematics in understanding material properties and behavior.

Ordinary and Partial Differential Equations in Material Science: Applying differential equations to model material phenomena.

Linear Algebra and Material Structures: Exploring the relationship between linear algebra and the structural properties of materials.

Case Studies and Applications: Practical applications and case studies demonstrating the use of mathematical methods in real-world material science problems.

CREDITS: 2

CATEGORY: Specialized Course

Mass & Heat Transfer

This course provides a comprehensive exploration of electrochemical energy systems, including batteries, fuel cells, supercapacitors, and electrolyzers. The course delves into the fundamental principles, design, and operation of these systems, with a strong emphasis on applications, performance, and technological advancements. Students will gain a solid foundation in both heat and mass transfer processes, crucial for understanding and optimizing energy systems.

Course Content.

Introduction to Heat and Mass Transfer: Basic principles and their relevance to energy systems.

Conduction Heat Transfer: Understanding the mechanisms and equations governing heat conduction.

Radiation Heat Transfer: Exploring thermal radiation principles and applications.

Fundamentals of Mass Transfer (Fick's Laws): Detailed study of diffusion processes and the governing laws.

Convective Mass Transfer: Examination of convective mass transfer principles and applications.

CREDITS: 2

CATEGORY: Specialized Course

Mathematical Modeling for Epidemiology

"Mathematical Modeling for Epidemiology" is a comprehensive course designed for graduate students to explore the use of mathematical models in understanding the spread and control of infectious diseases. This course covers the formulation, analysis, and interpretation of mathematical models in epidemiology, emphasizing both deterministic and stochastic approaches. Students will learn to apply these models to real-world scenarios, aiding in the prediction and control of epidemics.

Course Content.

Introduction to Epidemiology and Mathematical Modeling: This chapter provides an overview of epidemiology and the role of mathematical modeling in studying infectious diseases. Students will learn about basic epidemiological measures, such as incidence, prevalence, and the basic reproduction number (R_0), setting the foundation for more complex models.

Compartmental Models: Students will explore compartmental models, including the SIR (Susceptible-Infectious-Recovered) model and its extensions. The chapter covers the formulation of these models using differential equations, the interpretation of parameters, and the analysis of model dynamics. Variants such as SEIR (Susceptible-Exposed-Infectious-Recovered) and SIS (Susceptible-Infectious-Susceptible) models will also be discussed.

Deterministic Models: equilibrium points, stability analysis, and the use of phase plane methods. The chapter also covers the impact of interventions, such as vaccination and treatment, on the dynamics of infectious diseases.

Stochastic Models: The chapter covers basic stochastic processes, such as the Poisson process and Markov chains, and their application in modeling epidemics. Students will learn to simulate stochastic models and analyze their behavior using computational tools.

Spatial Models and Network Theory: metapopulation models, reaction-diffusion equations, and network-based models. Students will learn how to model disease spread on networks and analyze the impact of network topology on epidemic dynamics.

Parameter Estimation and Model Fitting: parameter estimation and model fitting, essential for applying mathematical models to real data. The chapter covers methods such as maximum likelihood estimation, Bayesian inference, and least squares fitting. Practical examples will illustrate how to fit models to epidemiological data and assess model accuracy.

Advanced Topics in Epidemiological Modeling: age-structured models, multi-host and multi-pathogen models, and models with heterogeneity in transmission. Students will explore the impact of population structure and behavioral changes on disease dynamics.

Applications and Case Studies: The course concludes with applications and case studies, demonstrating the use of mathematical models in understanding and controlling infectious diseases. Students will work on projects involving current and historical epidemics, such as COVID-19, influenza, and HIV/AIDS. These case studies will reinforce the concepts learned throughout the course and highlight the practical importance of epidemiological modeling.

CREDITS: 2

CATEGORY: Specialized Course

Mathematical Modeling of Cellular Dynamics

"Mathematical Modeling of Cellular Dynamics" is a comprehensive graduate-level course designed to explore the use of mathematical models to understand the complex dynamics of cellular processes. This course covers the formulation, analysis, and application of mathematical models to study interactions within cells, between cells, and with their environment. Emphasis is placed on both deterministic and stochastic modeling approaches, with applications to various biological processes, including cellular signaling, gene regulation, and disease progression.

Course Content.

Introduction to Cellular Dynamics and Mathematical Modeling: This chapter provides an overview of cellular dynamics and the importance of mathematical modeling in understanding cellular processes. Students will learn about key concepts in cellular biology, including cell structure, signaling pathways, and gene regulation, setting the foundation for more complex models.

Deterministic Models of Cellular Processes: explore deterministic models using ordinary differential equations (ODEs) and partial differential equations (PDEs) to describe cellular processes. Topics include modeling of biochemical reactions, enzyme kinetics, and signal transduction pathways. The chapter covers the formulation of these models, parameter estimation, and stability analysis.

Stochastic Models for Cellular Dynamics: stochastic differential equations (SDEs), the Gillespie algorithm, and Markov chains. Applications include modeling gene expression noise, variability in cellular responses, and the impact of random events on cellular behavior.

Spatial Modeling and Reaction-Diffusion Systems: Turing patterns, chemotaxis models, and intracellular transport. The chapter emphasizes the role of spatial heterogeneity in cellular dynamics.

Multi-Scale Modeling: This chapter introduces multi-scale modeling techniques that integrate cellular processes with tissue-level and organism-level dynamics. Students will learn how to link models of molecular interactions with larger-scale models of cellular and tissue behavior. Examples include integrating gene regulatory networks with models of tissue development.

Modeling Cellular Signaling Pathways: MAPK/ERK pathway, the Wnt signaling pathway, and the NF- κ B pathway. Topics include modeling feedback loops, signal amplification, and pathway cross-talk.

Gene Regulatory Networks and Synthetic Biology: network motifs, regulatory dynamics, and the design of synthetic genetic circuits. The chapter also explores the use of models to predict the behavior of engineered biological systems.

Applications and Case Studies: Students will work on projects involving current research topics such as cancer cell dynamics, immune cell interactions, and developmental biology. These case studies will reinforce the concepts learned throughout the course and highlight the practical importance of mathematical modeling in cellular biology.

CREDITS: 2

CATEGORY: Specialized Course

Mathematical Analysis of Models for Structured Population Dynamics

This is a graduate-level course that focuses on the use of mathematical models to understand the dynamics of structured populations. Structured population dynamics consider the variation in individual characteristics such as age, size, or spatial position within a population. This course covers the formulation, analysis, and application of mathematical models for structured populations, emphasizing both theoretical and practical aspects. Students will explore various models and techniques used to study the growth, interaction, and evolution of structured populations.

Course Content.

Introduction to Structured Population Dynamics: This chapter provides an overview of structured population dynamics and its importance in understanding population behavior. Students will learn about different types of structuring variables such as age, size, and spatial position, and how these structures affect population dynamics.

Age-Structured Models: McKendrick-von Foerster equation and its applications. Topics include the formulation of age-structured models using partial differential equations (PDEs), boundary conditions, and the interpretation of solutions. The chapter covers methods for analyzing the stability and long-term behavior of age-structured populations.

Size-Structured Models: formulation of size-structured models using PDEs and integro-differential equations. Topics include growth rates, reproduction rates, and mortality rates as functions of size, as well as techniques for analyzing the solutions of these models.

Spatially Structured Models: reaction-diffusion equations and their applications to population dynamics. The chapter covers the formulation of spatial models, including the incorporation of movement and dispersal mechanisms. Topics include pattern formation, traveling waves, and the impact of spatial heterogeneity on population dynamics.

Delay Differential Equations in Population Dynamics: This chapter introduces delay differential equations (DDEs) and their use in modeling populations with time delays. Students will learn about the formulation of DDEs, the interpretation of time delays in biological processes, and methods for analyzing the stability and dynamics of delay models.

Integral Equations and Structured Populations: formulation of integral equations for populations with continuous structuring variables, methods for solving these equations, and applications to various biological systems.

Numerical Methods for Structured Population Models: discretization methods for PDEs, DDEs, and integral equations, including finite difference methods, finite element methods, and numerical integration. The chapter emphasizes the implementation of these methods and the interpretation of numerical results.

Applications and Case Studies: Students will work on projects involving current research topics such as epidemiology, ecology, and conservation biology. These case studies will reinforce the concepts learned throughout the course and highlight the practical importance of structured population models.

CREDITS: 2

CATEGORY: Specialized Course

Computational Systems Biology

This course is designed to equip students with the computational techniques and models used to understand complex biological systems. The course covers the integration of data from different biological levels, from molecular to ecosystem, and the use of computational tools to simulate and analyze these systems. Topics include metabolic network modeling, regulatory network modeling, and whole-cell simulations.

Course Content.

Introduction to Systems Biology: This chapter introduces the key concepts and applications of systems biology. Students will learn about the holistic approach to studying biological systems, integrating data from genomics, proteomics, and metabolomics.

Metabolic Network Modeling: Students will explore the principles of metabolic network modeling, including stoichiometry, flux balance analysis (FBA), and constraint-based modeling. The chapter covers the formulation and analysis of metabolic networks, with hands-on exercises using computational tools.

Regulatory Network Modeling: Students will learn about the mathematical representation of regulatory networks, including Boolean networks, differential equation models, and stochastic models.

Whole-Cell Simulation: The chapter covers techniques for simulating entire cells, including multi-scale modeling and hybrid approaches that combine deterministic and stochastic methods.

Computational Tools and Software: Students will learn to use tools such as COBRA for metabolic modeling, CellDesigner for regulatory networks, and other relevant software.

Applications and Case Studies: Students will work on projects involving real-world biological systems, reinforcing the concepts learned throughout the course.

CREDITS: 2

CATEGORY: Specialized Course

Biostatistics and Data Analysis

"Biostatistics and Data Analysis" is a graduate-level course that provides a comprehensive introduction to statistical methods and data analysis techniques used in biological research. The course covers a range of topics from experimental design to advanced statistical methods and their application in analyzing biological data.

Course Content.

Introduction to Biostatistics: This chapter covers basic concepts in biostatistics, including types of data, summary statistics, and the importance of biostatistics in biological research. Students will learn about the role of biostatistics in designing experiments and analyzing data.

Experimental Design: randomization, replication, and blocking. The chapter covers different types of experimental designs such as completely randomized designs, randomized block designs, and factorial designs.

Descriptive Statistics: Students will learn about measures of central tendency, variability, and the use of graphical methods to describe data distributions.

Inferential Statistics: hypothesis testing, confidence intervals, and p-values. The chapter covers parametric and non-parametric tests, such as t-tests, chi-square tests, and rank-sum tests.

Advanced Statistical Methods: regression analysis, analysis of variance (ANOVA), and multivariate analysis. Students will learn about linear and logistic regression, mixed-effects models, principal component analysis (PCA), and cluster analysis.

VII. Applications and Case Studies: Students will work on projects involving real-world biological data, reinforcing the concepts learned throughout the course.

CREDITS: 2

CATEGORY: Specialized Course

Advanced Topics in Genomics and Proteomics

"Advanced Topics in Genomics and Proteomics" is a graduate-level course that explores the cutting-edge techniques and analytical methods used in genomics and proteomics. The course emphasizes the integration of high-throughput data and the computational approaches necessary to analyze and interpret these datasets.

Course Content.

Introduction to Genomics and Proteomics: This chapter provides an overview of genomics and proteomics, including key concepts, technologies, and applications. Students will learn about the importance of these fields in understanding biological processes and disease mechanisms.

Sequencing Technologies: next-generation sequencing (NGS), single-cell sequencing, and third-generation sequencing. The chapter covers the principles, workflows, and applications of these technologies in genomics research.

Data Analysis in Genomics: genome assembly, variant calling, functional annotation, and gene expression analysis. Students will learn to use bioinformatics tools and software for analyzing large-scale genomic datasets.

Proteomics Technologies: mass spectrometry, protein arrays, and protein-protein interaction networks. The chapter covers the principles and applications of these technologies in proteomics research.

Data Analysis in Proteomics: protein identification, quantification, and post-translational modifications. Students will learn to use proteomics software for data analysis and interpretation.

Integration of Omics Data: The chapter covers methods for multi-omics data integration and the challenges associated with these analyses.

Applications and Case Studies: Students will work on projects involving real-world omics data, reinforcing the concepts learned throughout the course.

CREDITS: 2

CATEGORY: Specialized Course

Systems Immunology

This course explores the use of systems biology approaches to understand the complex dynamics of the immune system. The course integrates experimental data with computational modeling to study immune responses at multiple scales, from molecular interactions to population-level effects. Students will gain a comprehensive understanding of immune system dynamics, modeling techniques, and their applications to immunological research.

Course Content.

Introduction to Systems Immunology: This chapter introduces the key concepts and significance of systems immunology. Students will learn about the holistic approach to studying the immune system, integrating data from genomics, proteomics, and metabolomics.

Modeling Immune Cell Dynamics: T-cell and B-cell interactions, antigen presentation, and cytokine signaling. The chapter covers the formulation and analysis of differential equation models to describe these processes.

Network-Based Approaches: immune network modeling, pathway analysis, and the use of graph theory to study immune interactions.

Data Integration: Students will delve into techniques for integrating high-throughput data with computational models. The chapter covers methods for combining genomic, proteomic, and metabolomic data to build comprehensive models of the immune system.

Applications: The course concludes with applications of systems immunology to various research areas. Students will work on projects involving autoimmune diseases, infectious diseases, and cancer immunotherapy, reinforcing the concepts learned throughout the course.

CREDITS: 2

CATEGORY: Specialized Course

Computational Neuroscience

This course is focused on the application of mathematical and computational techniques to understand the function of the nervous system. The course covers modeling at different levels, from individual neurons to neural networks and systems-level analysis. Students will gain a comprehensive understanding of neural modeling, network dynamics, and cognitive models.

Course Content.

Introduction to Computational Neuroscience: This chapter provides an overview of computational neuroscience, including its history and key concepts. Students will learn about the importance of computational approaches in understanding brain function and behavior.

Neural Modeling: Students will explore modeling techniques for individual neurons, including the Hodgkin-Huxley model, integrate-and-fire models, and biophysical models. The chapter covers the formulation and analysis of differential equation models to describe neural dynamics.

Synaptic Dynamics: Students will learn about synaptic dynamics, long-term potentiation (LTP), long-term depression (LTD), and spike-timing-dependent plasticity (STDP).

Network Dynamics: Students will delve into modeling neural networks, including small-world networks, neural coding, and information processing. The chapter covers techniques for analyzing network dynamics, such as mean-field theory and network oscillations.

Cognitive Models: This chapter introduces cognitive models, focusing on decision-making, learning and memory, and sensory processing. Students will explore computational models of cognitive processes and their neural correlates.

Applications: Students will work on projects involving brain-machine interfaces, neuroprosthetics, and computational psychiatry, reinforcing the concepts learned throughout the course.

CREDITS: 2

CATEGORY: Specialized Course

Evolutionary Systems Biology

In this course we explore how evolutionary processes shape biological systems. The course integrates principles from evolutionary biology with systems biology to study the dynamics of evolutionary change at the molecular, cellular, and organismal levels. Students will explore various models and techniques used to study evolutionary dynamics and their applications to biological research.

Course Content.

Introduction to Evolutionary Systems Biology: Students will learn about the integration of evolutionary biology with systems biology to study complex biological systems.

Modeling Evolutionary Dynamics: population genetics, adaptive landscapes, and fitness landscapes. The chapter covers the formulation and analysis of differential equation models to describe evolutionary processes.

Gene Regulatory Networks: Students will learn about network motifs, regulatory dynamics, and the impact of mutations on network function.

Evolution of Metabolic Networks: Students will delve into the evolution of metabolic networks, examining adaptation and optimization of metabolic pathways. The chapter covers techniques for modeling metabolic evolution and analyzing the impact of environmental changes on metabolic networks.

Experimental Evolution: Students will learn about techniques for conducting evolutionary experiments and analyzing the resulting data.

Applications: The course concludes with applications of evolutionary systems biology to various research areas. Students will work on projects involving the evolution of drug resistance, metabolic engineering, and synthetic biology, reinforcing the concepts learned throughout the course.

CREDITS: 2

CATEGORY: Specialized Course

Mean Field Games and Collective Behaviour

This course covers the fundamental concepts, analytical techniques, and numerical methods used to analyze and predict the behavior of large populations of interacting agents. Students will explore applications in economics, social sciences, biology, and engineering.

Course Content.

Introduction to Mean Field Games and Collective Behaviour: Overview of mean field game theory. Key concepts in collective behavior. Historical development and interdisciplinary applications.

Mathematical Foundations of Mean Field Games: Basic principles of game theory. Hamilton-Jacobi-Bellman (HJB) equations. Fokker-Planck-Kolmogorov (FPK) equations. Coupled HJB-FPK systems.

Analytical Techniques for Mean Field Games: Existence and uniqueness of solutions. Stationary and time-dependent problems. Linear and nonlinear mean field games. Mean field equilibrium concepts.

Numerical Methods for Mean Field Games: Finite difference methods. Spectral methods. Monte Carlo simulations. High-dimensional approximation techniques.

Collective Behaviour Models: Swarming and flocking models. Opinion dynamics. Epidemic modeling and spreading processes. Traffic and crowd dynamics.

Agent-Based Modeling and Simulations: Principles of agent-based models. Implementing agent-based simulations. Comparison with mean field models. Applications in social and biological systems.

Applications of Mean Field Games and Collective Behaviour: Economic and financial markets. Networked systems and cyber-physical systems. Biological systems and ecosystems. Engineering and robotics.

Advanced Topics in Mean Field Games: Stochastic mean field games. Mean field games with constraints. Learning in mean field games. Multi-population mean field games.

CREDITS: 2

CATEGORY: Specialized Course

Control of Distributed Systems

Distributed systems are characterized by multiple interconnected components that may be spatially distributed. The course covers theoretical foundations, modeling techniques, control strategies, and practical implementations of distributed control systems in various engineering fields.

Course Content.

Introduction to Distributed Systems: Definition and characteristics of distributed systems. Examples and applications in engineering. Challenges in controlling distributed systems.

Mathematical Modeling of Distributed Systems: Partial differential equations (PDEs). Difference equations for discrete systems. State-space representation of distributed systems. Model reduction techniques.

Fundamentals of Distributed Control: Principles of distributed control. Decentralized vs. distributed control. Control architectures: hierarchical, distributed, and hybrid.

Control Design Techniques: Feedback control for distributed systems. Optimal control methods. Robust control strategies. Distributed optimization techniques.

Communication in Distributed Systems: Communication protocols and networks. Impact of communication delays and losses. Networked control systems. Consensus algorithms.

Advanced Topics in Distributed Control: Distributed Kalman filtering. Event-triggered and self-triggered control. Multi-agent systems and cooperative control. Adaptive and learning-based distributed control.

Applications and Case Studies: Smart grids and energy systems. Distributed robotics and autonomous systems. Environmental monitoring and control. Industrial process control.

Implementation and Simulation: Simulation tools and software for distributed control. Real-time implementation challenges. Case studies and practical projects.

CREDITS: 2

CATEGORY: Specialized Course

PDE-Constrained Games and Emerging Engineering Applications

This graduate-level course explores the intersection of partial differential equations (PDEs) and game theory, focusing on their applications in engineering. The course covers the mathematical formulation of PDE-constrained games, solution techniques, and the implementation of these models in emerging engineering applications. Students will gain a deep understanding of how to use PDEs in the context of strategic interactions and optimization problems.

Course Content.

Introduction to PDE-Constrained Games: Overview of PDE-constrained games. Applications in engineering and other fields. Differences between traditional games and PDE-constrained games.

Mathematical Foundations: Review of partial differential equations. Basics of game theory and Nash equilibrium. Coupling PDEs with game-theoretic models.

Formulation of PDE-Constrained Games: Problem setup and constraints. Formulating cost functions and payoffs. Boundary and initial conditions in PDEs.

Analytical Techniques: Existence and uniqueness of solutions. Optimal control theory. Variational methods and saddle point problems.

Numerical Methods for PDE-Constrained Games: Finite element methods. Finite difference methods. Spectral methods. Iterative solvers and convergence analysis.

Applications in Emerging Engineering Fields: Smart grids and energy management. Traffic flow and transportation systems. Environmental engineering and resource management. Robotics and autonomous systems.

Advanced Topics: Stochastic PDE-constrained games. Mean field game theory. Learning and adaptive strategies in PDE-constrained settings. Multi-agent systems and cooperative control.

Implementation and Case Studies: Simulation tools and software for PDE-constrained games. Real-world case studies and applications. Practical projects involving engineering applications.

CREDITS: 2

CATEGORY: Specialized Course

Numerical Control of Waves

This graduate-level course focuses on the numerical methods and strategies used to control wave propagation in various physical and engineering systems. The course covers the mathematical formulation of wave control problems, the development and implementation of numerical techniques, and the application of these methods to practical engineering problems involving waves in fluids, solids, and electromagnetic fields.

Course Content.

Introduction to Wave Control: Overview of wave phenomena in engineering and physics. Importance and applications of wave control. Challenges in controlling wave propagation.

Mathematical Foundations: Review of wave equations (linear and nonlinear). Boundary and initial conditions. Control theory basics: controllability and observability.

Formulation of Wave Control Problems: Setting up control objectives. Boundary control and internal control methods. Cost functionals and optimization criteria.

Numerical Methods for Wave Control: Finite difference methods for wave equations. Finite element methods for wave control. Spectral methods and their applications. Time integration schemes and stability analysis.

Control Techniques: Exact controllability methods. Optimal control theory applied to wave propagation. Feedback control strategies. Robust and adaptive control techniques.

Applications in Engineering: Control of acoustic waves. Control of elastic waves in structures. Electromagnetic wave control in antennas and waveguides. Fluid dynamics and control of water waves.

Advanced Topics: Inverse problems in wave control. Stochastic wave control problems. Nonlinear wave control. Multi-scale and multi-physics wave control.

Implementation and Case Studies: Software tools and simulation environments for wave control. Practical examples and case studies in engineering applications. Project work involving real-world wave control problems.

CREDITS: 2

CATEGORY: Specialized Course

Robotic Path Planning and Task Execution

In this course, we explore the algorithms and techniques used for planning and executing paths and tasks in robotic systems. The course covers the mathematical foundations, computational methods, and practical applications of robotic path planning and task execution in various environments. Special emphasis is placed on partial differential equations (PDEs) such as the eikonal equation, which are critical in many advanced path planning algorithms. Students will learn about different planning algorithms, optimization methods, and implementation strategies for robotic systems.

Course Content.

Introduction to Robotic Path Planning: Overview of robotic path planning. Applications in industry, healthcare, and autonomous vehicles. Challenges in robotic path planning.

Mathematical Foundations: Configuration space and state space representation. Graph theory and search algorithms. Optimization techniques in path planning.

Basic Path Planning Algorithms: Grid-based methods: Dijkstra's algorithm, A^* algorithm. Sampling-based methods: Rapidly-exploring Random Trees (RRT), Probabilistic Roadmaps (PRM). Potential field methods.

Partial Differential Equations in Path Planning: Introduction to PDEs relevant to path planning. The eikonal equation and its applications in robotics. Fast marching method and level set methods. Solving PDEs for optimal path planning.

Advanced Path Planning Techniques: Motion planning for dynamic environments. Optimization-based methods: Mixed-integer programming, convex optimization. Heuristics and metaheuristics for complex path planning problems.

Task Planning and Execution: Task scheduling and allocation. Multi-robot task planning. Integration of task and path planning.

Learning-Based Methods: Reinforcement learning for path planning. Supervised and unsupervised learning techniques. Neural networks and deep learning applications.

Applications and Case Studies: Robotic path planning in manufacturing and logistics. Autonomous navigation in unmanned aerial vehicles (UAVs). Medical robotics and assistive technologies.

Simulation and Implementation: Simulation tools and environments for robotic path planning. Real-time path planning and execution. Case studies and project-based learning.

CREDITS: 2

CATEGORY: Specialized Course

Modeling of Autonomous Systems

This course delves into the principles, techniques, and applications of modeling autonomous systems. The course covers the mathematical foundations, computational methods, and practical applications of modeling autonomous vehicles, drones, and robotic systems. Special emphasis is placed on dynamic modeling, system identification, and simulation techniques necessary for the design and analysis of autonomous systems. Students will learn to create accurate models that predict the behavior of these systems in various environments.

Course Content.

Introduction to Autonomous Systems: Overview of autonomous systems and their applications. Historical development and current trends. Challenges in modeling autonomous systems.

Mathematical Foundations: Coordinate systems and transformations. Kinematics and dynamics of rigid bodies. State-space representation.

Dynamic Modeling Techniques: Modeling of ground vehicles, aerial vehicles, and marine vehicles. Newton-Euler and Lagrange methods. Modeling of multi-body systems.

System Identification: Techniques for system identification. Time-domain and frequency-domain identification. Parameter estimation methods.

Control-Oriented Modeling: Linear and nonlinear system modeling. Model reduction techniques. Control system design based on dynamic models.

Simulation of Autonomous Systems: Simulation environments and tools (e.g., MATLAB/Simulink, ROS, Gazebo). Real-time simulation and hardware-in-the-loop (HIL) simulation. Validation and verification of models.

Advanced Topics in Modeling: Hybrid systems modeling. Uncertainty quantification and robustness analysis. Learning-based modeling approaches (e.g., neural networks, reinforcement learning).

Applications and Case Studies: Autonomous ground vehicles (e.g., self-driving cars). Unmanned aerial vehicles (UAVs). Autonomous marine vessels. Industrial and service robots.

Project and Implementation: Hands-on projects involving modeling and simulation of autonomous systems. Case studies from industry and research. Presentation and documentation of project results.

CREDITS: 2

CATEGORY: Specialized Course

Robotic Behaviors and Odometry

"Robotic Behaviors and Odometry" is a graduate-level course that focuses on the fundamental behaviors of robotic systems and the methods used to estimate their position and orientation over time. The course covers the mathematical foundations, algorithms, and practical implementations of basic robotic behaviors such as navigation, obstacle avoidance, and sensor integration. Special emphasis is placed on odometry, which is crucial for accurate localization and navigation in robotic systems. Students will learn about different odometry techniques, sensor fusion, and implementation strategies for autonomous robotic systems.

Course Content.

Introduction to Basic Robotic Behaviors: Overview of basic robotic behaviors. Applications in industry, healthcare, and autonomous vehicles. Challenges in implementing robotic behaviors.

Mathematical Foundations: Coordinate systems and transformations. Kinematics of mobile robots. Sensor modeling and calibration.

Odometry Techniques: Wheel encoders and dead reckoning. Inertial measurement units (IMUs). Visual odometry. Sensor fusion for improved odometry.

Localization and Mapping: Introduction to localization and mapping. Extended Kalman Filter (EKF) for localization. Particle filter localization. Simultaneous Localization and Mapping (SLAM).

Basic Robotic Behaviors: Reactive behaviors: obstacle avoidance, wall-following. Deliberative behaviors: path planning, goal seeking. Hybrid behaviors: combining reactive and deliberative strategies.

Navigation Algorithms: Grid-based navigation. Potential fields. Vector field histogram (VFH). Dynamic window approach (DWA).

Advanced Sensor Integration: Integration of LiDAR, sonar, and camera sensors. Data fusion techniques for robust perception. Real-time sensor processing.

Applications and Case Studies: Robotic navigation in indoor and outdoor environments. Service robots in healthcare and hospitality. Industrial robots in manufacturing and logistics.

Simulation and Implementation: Simulation tools and environments for robotic behaviors and odometry. Real-time implementation and testing. Case studies and project-based learning.

CREDITS: 2

CATEGORY: Specialized Course